# Resource Pricing for Fair and Efficient Link Sharing with a Finite Data Model

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*Abstract*— **The literature on pricing implicitly assumes an "infinite data" model, in which sources can sustain any data rate indefinitely. We assume a more realistic "finite data" model, in which sources occasionally run out of data. Further, we assume that users have contracts with the service provider, specifying the rates at which they can inject traffic into the network. Our objective is to study how prices can be set such that a single link can be shared efficiently and fairly among users in a dynamically changing scenario where a subset of users occasionally has little data to send. We obtain simple necessary and sufficient conditions on prices such that efficient and fair link sharing is possible. We illustrate the ideas using a simple example.**

## I. Introduction

Pricing has been suggested as a mechanism to control congestion and ensure fair and efficient operation of networks. In much of the published literature ([1] [2] [3] [4] [5], [6] [7]), elastic traffic is considered and the context of operation is as follows. Each user of the network has a utility function that quantifies the benefit that she derives from the network. The utility function is an increasing (often, concave increasing) function of the rate at which the user can send data through the network. The system objective is maximization of the sum of all users' utility functions. The problem is to find the vector of users' rates such that the system objective is realized.

The resulting constrained optimization problem can be solved in a centralized manner if all the utility functions were known. In practice, however, there is no central authority that computes rates and further, the users' utility functions are not known. In [1], Kelly proposed a decentralized method to arrive at the system-optimal rates. In this method, the network declares *prices*, and each user individually solves the problem of maximizing her *net benefit* or *net utility*, which is her utility minus the total cost paid to the network. He shows that there exists a vector of prices such that the vector of individually optimal rates arrived at by the users is, indeed, the system-optimal rate vector.

However, the published literature tacitly assumes an *infinite data model*. Every source is assumed to have an infinite backlog of data. The implication is that a source can send traffic at *any* rate (obtained from the solution to the individual optimization problem) continuously — there is never a dearth of data. In practice, of course, sources will occasionally run out of data. We consider a *finite* data model, in which, occasionally, a source does not have enough data to send. Therefore, a source may not be able to sustain a data rate that is suitable for a fair and efficient operation of the system.

Further, in practice, users have contracts with the service provider (SP) that specify the rates at which they can send traffic into the network. Hence, the practical problem is often *not* to find out the correct vector of rates by using a distributed algorithm—the correct vector is simply the vector of contracted rates. Rather, it is important to devise a scheme of operation such that any slack caused by a user who is sending traffic at a rate lower than her contracted rate—referred to as an "underuser" henceforth—can be utilized by others. Correspondingly, a user with plenty of data available is referred to as an "overuser" because she can send data at a rate higher than her contracted rate.

We also believe that one must have congestion-dependent and **user-dependent** pricing. If the network is not congested, then the price should remain low, so that users with excess data can utilize the network. But when the network becomes congested, the price should not increase equally for all users; rather, those users that have exceeded their contracted rates and have caused congestion should be charged heavily, while those who are compliant should be charged at no more than their nominal rates.

Summarizing, our approach is different from that in the literature in the following respects: *(a)* finite data model, *(b)* contracted rates and *(c)* congestion-dependent as well as user-dependent pricing.

We are interested in ensuring that network operation is characterised by the following.

- When some users are underusers because of limited available data, it should be possible for others to increase their rates so as to utilize the slack. Does there exist a pricing scheme such that users with plenty of data available are *encouraged* to become overusers? This means that in this situation, these users' net utilities should be maximized at values higher than the corresponding contracted rates.
- Later, when underusers wish to increase their rates because they have more data to send, they should have the incentive to do so and overusers should be encouraged to back down. Does their exist a pricing scheme such that this happens? Again, this means that the users' net utility values should be maximized at the appropriate points.

In [8], [9] and [10], the authors consider priority queueing to provide differentiated services to a mix of elastic and real-time traffic. Users choose the priority class to which their traffic belongs. Higher priority traffic experiences better service but its price is higher. Game-theoretic analysis is used to investigate whether a system equilibrium exists. [8] also considers how the network operator can set prices such that revenue is maximized at equilibrium. In our work, we do not have multiple classes and we do not consider priority queueing. There is only one traffic class, carrying elastic traffic.

## II. Model

We consider a single link which is shared among multiple users. Our model consists of the following elements.

- $N$ users sharing a link; $N$ is a given and fixed integer.
- The capacity of the link is $C$ bits/sec.
- We use a fluid model for traffic. User $i$ generates fluid at rate $\lambda_i$. We emphasize that this is a variable, because the user may not be able to generate traffic at a constant rate throughout.
- User $i$ has a contract to send traffic at rate $\gamma_i$, and the price charged by the SP is $\pi_i$; this is the *total price*, not the price per unit flow. When $\lambda_i > \gamma_i$, we call $(\lambda_i - \gamma_i)$ the "excess rate." Throughout this paper, we assume that the sum of the contracted rates equals the link capacity, *i.e.*,

$$\sum_{i=1}^{N} \gamma_i = C$$

- The *utility* of user $i$ is defined to be the rate of user $i$ traffic actually carried by the SP. Suppose that a fraction $\beta$ of user $i$'s traffic is dropped. Then the utility for user $i$ is $\lambda_i(1-\beta)$.
- When user $i$ is not able to send traffic at her contracted rate $\gamma_i$, we consider a "disutility" for user $i$. This measures the amount of dissatisfaction that user $i$ suffers from at not being able to generate sufficient traffic. The disutility function is defined to be zero when $\lambda_i \geq \gamma_i$. The specific disutility function we use is $e^{(\gamma_i - \lambda_i)}$. It may be noted that the disutility increases rapidly as $\lambda_i$ drops below $\gamma_i$. The disutility term encourages underusers to increase their rates whenever possible.
- Finally, the "net utility" of user $i$ is defined to be her utility minus disutility (which may be zero) minus the price paid to the SP.

### A. Pricing Scheme

The pricing scheme is characterized by the following features.

- When the resource is not congested:
  - Underusers are charged less than their contracted price; if a user is sending at a rate which is a fraction $f$ of her contracted value, the charge is correspondingly a fraction $f$ of her contracted charge.
  - Overusers are charged their contracted prices.

The rationale for this is that as long as there is no congestion, users should be permitted to go above their contracted rates.

- When the resource is congested:
  - Underusers are again charged correspondingly lower prices.
  - Overusers are charged heavily; if user $i$ is sending at a rate $\lambda_i > \gamma_i$, then the price charged is $\pi_i e^{(\lambda_i - \gamma_i)}$. Thus, the price increases steeply with excess rate.

In Figure 1, we give a schematic representation of the pricing scheme. For simplicity, we assume a point $C$ such that when the aggregate traffic using the resource is less than or equal to $C$, the resource is not congested. When the resource is a link, $C$ is simply the link capacity. When the aggregate rate into the link is more than $C$, the link is congested and part of the traffic is lost. When user 1 transmits at a rate less than or equal to her contracted value $\gamma_1$, the price charged is less than or equal to the contracted price $\pi_1$, *irrespective of whether the resource is congested or not* (the curves corresponding to $\lambda_1 < \gamma_1$, and $\lambda_1 = \gamma_1$). Thus, a user who complies with the contract and does not contribute to the congestion is not penalized. However, when user 1 transmits at a rate greater than $\gamma_1$, the price charged depends on the level of congestion — without congestion, the charge is $\pi_1$, but when the link is congested, the price is considerably higher (the curve corresponding to $\lambda_1 > \gamma_1$). Thus, when there is no congestion, a user is allowed to send at a rate higher than her contracted value at no extra cost; but when congestion occurs, the user is charged heavily to induce her to reduce the rate.

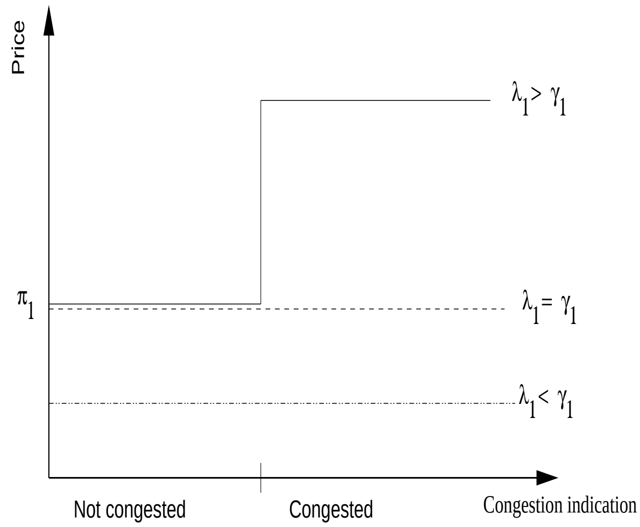


Fig. 1. Schematic showing how the price charged by the SP changes, depending on both the rate at which a user sends traffic and the state of congestion of the resource.

### B. Fair and Efficient Link Sharing

Our intention is to understand how the prices $\pi_i$, $1 \leq i \leq N$, can be set, so that the system behaviour is "desirable" in the following sense. Let $\mathcal{L}$ and $\mathcal{H}$ denote the set of underusers and overusers, respectively.

1. When the resource is not congested because users in $\mathcal{L}$

are below their contracted rates, other users in the complement $\mathcal{L}^c$ are not prevented from going above their contracted rates. This is desirable so that the resource is fully utilized.

2. Suppose that the resource is fully utilized and, further, that there is no congestion. Now if users in $\mathcal{L}$ wish to increase their rates and go up to their contracted values, they should have the incentive to do that; at the same time, users in $\mathcal{H}$ should have the incentive to reduce their rates. This is desirable so that users in $\mathcal{L}$ can reclaim their share of the network resources from users in $\mathcal{H}$ when they wish to do so.

## III. Analysis

In this section, we explore how the prices $\pi_i$, $1 \le i \le N$, can be set so that desirable behaviour occurs. We focus on user $i$ and assume that the transmission rates $\lambda_j$ of the other users $j, 1 \le j \le N, j \ne i$, are given and fixed.

Due to space constraints, we provide only a proof-outline for one of the following results in the Appendix. In all cases, the proofs are based on the definition of net utility for that case, elementary calculus and simple algebra.

### A. Other users below aggregate contracted rate

In this section, we consider a system where $\sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j < C - \gamma_i$. In such a system, user $i$ should have the incentive to transmit at a rate $(C - \sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j)$, so that the link is fully utilized, but without causing congestion. We wish to find out how to set the price $\pi_i$ such that this goal is achieved.

The net utility function for user $i$ is denoted as $NU_i$. If $\frac{dNU_i}{d\lambda_i} > 0$, then $i$ has incentive to increase her rate beyond $\lambda_i$. Corresponding conclusions apply when the derivative at $\lambda_i$ is negative or zero.

*Lemma 3.1:* When the total traffic from users $j \in \{1, 2, \ldots, N\}$, $j \ne i$, is less than their aggregate contracted rate, $NU_i$ is maximized at the point $\lambda_i^* = (C - \sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j)$ if and only if the price $\pi_i$ satisfies

$$\left(\frac{\sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j}{C}\right) e^{\sum_{\substack{j=1\\j\ne i}}^{N} (\lambda_j - \gamma_j)} \le \pi_i \le 2\gamma_i \qquad (1)$$

Further, $NU_i$ is an increasing function of $\lambda_i$ to the left of $\lambda_i^*$ and a decreasing function of $\lambda_i$ to the right of $\lambda_i^*$.

### B. Other users at aggregate contracted rate

Here we have $\sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j = C - \gamma_i$. User $i$ needs to transmit at a rate $\gamma_i$ so that the link is fully utilized and yet there is no congestion.

*Lemma 3.2:* When the total traffic from users $j \in \{1, 2, \ldots, N\}$, $j \ne i$, is equal to their aggregate contracted rate, $NU_i$ is maximized at the point $\lambda_i^* = \gamma_i$ if and only if the price $\pi_i$ satisfies

$$\frac{C - \gamma_i}{C} \le \pi_i \le 2\gamma_i \qquad (2)$$

Further, $NU_i$ is an increasing function of $\lambda_i$ to the left of $\lambda_i^*$ and a decreasing function of $\lambda_i$ to the right of $\lambda_i^*$.

We note that the interval in Equation 2 is a subset of that in Equation 1 because the lower bounds are ordered. So, a choice of $\pi_i$ satisfying Equation 2 will always satisfy Equation 1.

### C. Other users above aggregate contracted rate

Because the other users are sending an aggregate amount of traffic that exceeds the sum of their contracted rates, we have $\sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j > C - \gamma_i$. In this case, it is desirable that, notwithstanding the congested link, user $i$ be able to increase her rate to her "rightful" share $\gamma_i$ while other users back down.

*Lemma 3.3:* When the total traffic from users $j \in \{1, 2, \ldots, N\}$, $j \ne i$, is more than their aggregate contracted rate, $NU_i$ is maximized at the point $\lambda_i^* = \gamma_i$ if and only if the price $\pi_i$ satisfies

$$\frac{C \sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j}{(\gamma_i + \sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j)^2} \le \pi_i \le \gamma_i \left( \frac{C \sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j}{(\gamma_i + \sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j)^2} + 1 \right) \qquad (3)$$

Further, $NU_i$ is an increasing function of $\lambda_i$ to the left of $\lambda_i^*$ and a decreasing function of $\lambda_i$ to the right of $\lambda_i^*$.

We note that the upper limit in Equation 3 will always be strictly less than $2\gamma_i$. So, the intersection of all three intervals has a right edge strictly smaller than $2\gamma_i$.

Having obtained the three ranges of $\pi_i$, we would like to know what the intersection of the three ranges looks like. The significance of this is that it may be possible to choose a value of $\pi_i$ that will work (*i.e.,* lead to desirable behaviour) irrespective of whether the aggregate traffic from users $j \in \{1, 2, \ldots, N\}, j \ne i$, is below, at or above the corresponding aggregate contracted rate.

*Lemma 3.4:* The intersection of the three ranges for $\pi_i$ coincides with the interval in Equation 3 if and only if

$$\sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j \le \frac{{\gamma_i}^2}{C - \gamma_i} \qquad (4)$$

Else, the interval is $\left( \frac{C-\gamma_i}{C}, \gamma_i \left( \frac{C \sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j}{(\gamma_i + \sum_{\substack{j=1\\j\ne i}}^{N} \lambda_j)^2} + 1 \right) \right)$

Thus, for each $i$, we check if Equation 4 holds or not. The answer to this question determines the "intersection interval."

## IV. Discussion

We start with the following question: For each of the three intervals in Equations 1, 2 and 3, is it true that the left edge is strictly smaller than the right edge?

The left edge in the first case is smaller than the left edge in the second case. Also, the right edges in the two cases are the same. So, it is enough to ensure that the second case is a valid interval. Simple algebra shows that the condition is

$$\frac{C}{\gamma_i} < 2C + 1$$

A sufficient condition for the interval in Equation 3 to be a valid interval is $\gamma_i \ge 1$. If $\gamma_i \ge 1$, then we see immediately that the previous sufficient condition is satisfied too. Thus, $\gamma_i \ge 1$, for $i = 1, 2, \ldots, N$, is a condition that is sufficient for the intervals in all three cases to be valid. Essentially, the unit of rate should be chosen to be the *smallest* contracted rate value among the $N$ connections.

Consider those values of $i$ such that Equation 4 holds. For these values of $i$, Lemma 3.4 says that the intersection interval coincides with the interval in Equation 3. Therefore, taking $\gamma_i \geq 1$ for all $i$ already ensures that we have valid intervals for these $i$ values.

For values of $i$ such that Equation 4 is false, we know from Lemma 3.4 that the intersection interval is $\left(\frac{C-\gamma_i}{C}, \gamma_i\left(\frac{C\sum_{j=1, j\neq i}^{N}\lambda_j}{(\gamma_i+\sum_{j=1, j\neq i}^{N}\lambda_j)^2}+1\right)\right)$. Now $\gamma_i \geq 1$ is sufficient for this interval to be valid also, because in that case, the left edge $\frac{C-\gamma_i}{C} < 1$, while the right edge $\gamma_i\left(\frac{C\sum_{j=1, j\neq i}^{N}\lambda_j}{(\gamma_i+\sum_{j=1, j\neq i}^{N}\lambda_j)^2}+1\right)$ is larger than 1.

Hence, $\gamma_i \geq 1$ for $i = 1, 2, \ldots, N$, ensures that the intersection interval for each $i$ is a valid interval, and hence $\pi_i$ can be chosen to enforce desirable behaviour.

It can be seen that when prices are set according to the interval $\left(\frac{C\sum_{j=1, j\neq i}^{N}\lambda_j}{(\gamma_i+\sum_{j=1, j\neq i}^{N}\lambda_j)^2}, \gamma_i\left(\frac{C\sum_{j=1, j\neq i}^{N}\lambda_j}{(\gamma_i+\sum_{j=1, j\neq i}^{N}\lambda_j)^2}+1\right)\right)$ (Equation 3) or $\left(\frac{C-\gamma_i}{C}, \gamma_i\left(\frac{C\sum_{j=1, j\neq i}^{N}\lambda_j}{(\gamma_i+\sum_{j=1, j\neq i}^{N}\lambda_j)^2}+1\right)\right)$ (as mentioned in Lemma 3.4), $(\gamma_1, \gamma_2, \ldots, \gamma_N)^t$ is a Nash equilibrium. Considering any $i \in \{1, 2, \ldots, N\}$, we find that the aggregate rate from the other users is equal to the sum of their contracted rates, and therefore, Lemma 3.2 applies. This means that user $i$ has no incentive to change her rate from $\gamma_i$, and this conclusion applies to all the users. Hence, $(\gamma_1, \gamma_2, \ldots, \gamma_N)^t$ is a Nash equilibrium when prices are set according to the intervals mentioned in Lemma 3.4.

Moreover, it can be seen that $(\gamma_1, \gamma_2, \ldots, \gamma_N)^t$ is the unique Nash equilibrium in this problem. We consider a $\overline{\lambda}$ with $\sum_{j=1}^{N}\lambda_j = C$, but with one $i$ for which $\lambda_i > \gamma_i$. Then, there must be a $k$ for which $\lambda_k < \gamma_k$. With respect to this $k$, $\sum_{j=1, j\neq k}^{N}\lambda_j > \sum_{j=1, j\neq k}^{N}\gamma_j$, and the third case above applies. Therefore, by Lemma 3.3, $k$ would like to increase her rate to $\gamma_k$. Hence $\overline{\lambda}$ is not a Nash equilibrium. In other cases as well ($\sum_{j=1}^{N}\lambda_j > C$ and $\sum_{j=1}^{N}\lambda_j < C$) we can argue similarly. Hence, $(\gamma_1, \gamma_2, \ldots, \gamma_N)^t$ is the unique Nash equilibrium here.

## V. Experimental Results with Multiple Users accessing a Single Link

The scenario of multiple users accessing a single link is simulated using MATLAB 7.0. The link capacity $C$ is assumed to be 5 units and the number of users $N = 4$. The vector of contracted rates is taken to be [1 1 1 2] in this experiment. We observe that the contracted rates of all the users add up to the capacity of the link. An initial rate vector $\lambda$ is chosen in which some users are above their contracted rates and some others are below theirs. For this four-user system we take $\lambda$ = [1.5 2 0.5 1.5]. So users 1 and 2 are overusers and users 3 and 4 are underusers. This means that at the start of the simulation, some users have limited data to send and hence they are not able to utilize their contracted rates fully. Just after the start, we assume that the limited data condition is removed, and each user now has sufficient data to sustain any data rate. Each user explores the neighbourhood of her present rate value to see if her net utility can be improved. In the MATLAB experiment, we use a step size of 0.01.

The initial prices $\pi_i$, $1 \leq i \leq 4$, are calculated as follows. For user 1, let $\theta$ be the sum of the rates of all users except user 1. So $\theta = 2+0.5+1.5 = 4$, and $\frac{\gamma_1^2}{(C-\gamma_1)} = \frac{1}{(5-1)} = 0.25$. Since $\theta > \frac{\gamma_1^2}{(C-\gamma_1)}$, the range for $\pi_1$ is

$$\left(\frac{C-\gamma_1}{C}\right) \leq \pi_1 \leq \gamma_1\left(\frac{C\theta}{(\gamma_1+\theta)^2}+1\right)$$

which reduces to

$$0.8 \leq \pi_1 \leq 1.8$$

We assign a value in the middle of this range to $\pi_1$. So $\pi_1 = 1.3$.

In the same way, we obtain the price-ranges for users 2, 3 and 4. We get

$$0.8 \leq \pi_2 \leq 1.8642$$
$$0.8 \leq \pi_3 \leq 1.6944$$
$$0.6 \leq \pi_4 \leq 3.1120$$

and we choose $\pi_2 = 1.3321$, $\pi_3 = 1.2472$ and $\pi_4 = 1.8560$.

On using these price-values, we observe that each user converges to her contracted rate in the process of maximizing her respective net utility. Figure 2 illustrates these results.

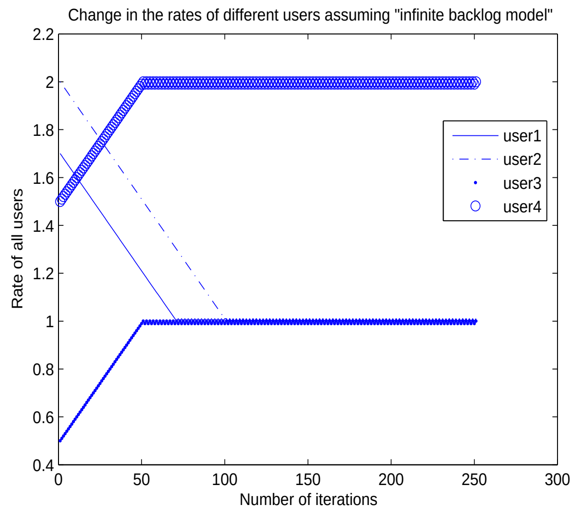


Fig. 2. Just before the start, $\lambda$ = [1.5 2 0.5 1.5]. At the start of the simulation, all users have sufficient data to sustain any rate. The pricing scheme ensures that the rates of all the users converge to their respective contracted values.

Now suppose user 1 and 2 run out of data and their rates drop to 0.5 and 0.7, respectively. We would like users 3 and 4 to use up the spare capacity left by users 1 and 2. At this point, $\lambda$= [0.5 0.7 1 2]. For user 3, $\theta = 0.5+0.7+2 = 3.2$, and $\frac{\gamma_3^2}{(C-\gamma_3)} = \frac{1}{(5-1)} = 0.25$. Since $\theta > \frac{\gamma_3^2}{(C-\gamma_3)}$, the range for $\pi_3$ is found to be

$$0.8 \leq \pi_3 \leq 1.9070$$

So the original $\pi_3(= 1.2472)$ is still inside the valid range. A similar calculation for user 4 shows that

$$0.6 \leq \pi_4 \leq 3.2472$$

and the original $\pi_4(= 1.8560)$ remains within the above range. Thus, the same prices can be retained in this situation.

In the simulation, we keep user 1 and 2 fixed at 0.5 and 0.7 respectively and allow user 3 and 4 to keep checking their net utilities at each iteration and try to maximize it as described previously. We observe that user 3 increases her rate from 1 to 1.4 and user 4 increases hers from 2 to 2.4, thus equally sharing the extra bandwidth between themselves. Figure 3 shows these results.

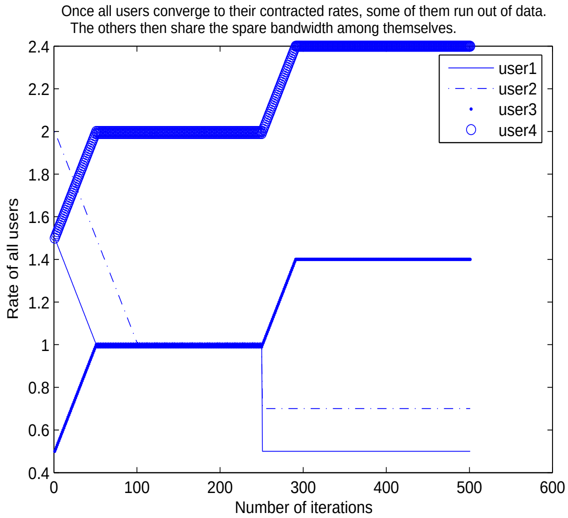


Fig. 3. Just before time 250, all users are at their contracted values. At time 250, users 1 and 2 are limited to sending traffic at rates 0.5 and 0.7, respectively. The pricing scheme ensures that users 3 and 4 are able to go beyond their contracted rates and utilize the link fully.

Now let us assume that user 1 and 2 again have large amounts of data to send, and can sustain any rate. We would like to find out whether they can reclaim their share of bandwidths again. At this point, $\lambda = [0.5\ 0.7\ 1.4\ 2.4]$. For user 1 and 2 to increase their rates, the prices should be readjusted if necessary. Now we check if any readjustment is needed or not.

Similar calculations as before indicate the following ranges.

$$0.8 \leq \pi_1 \leq 1.7438$$

$$0.8 \leq \pi_2 \leq 1.7654$$

$$0.8 \leq \pi_3 \leq 1.8507$$

$$0.6 \leq \pi_4 \leq 4.0062$$

and hence it may be noted that the original $\pi_i$ values are still within these new ranges above.

Therefore, the prices need not be changed for user 1 and 2 to reclaim their bandwidth shares. In the simulation, all users monitor their respective net utilities at each iteration and try to maximize it. We find that at equilibrium, all users again converge to their respective contracted rates. Figure 4 shows these results.

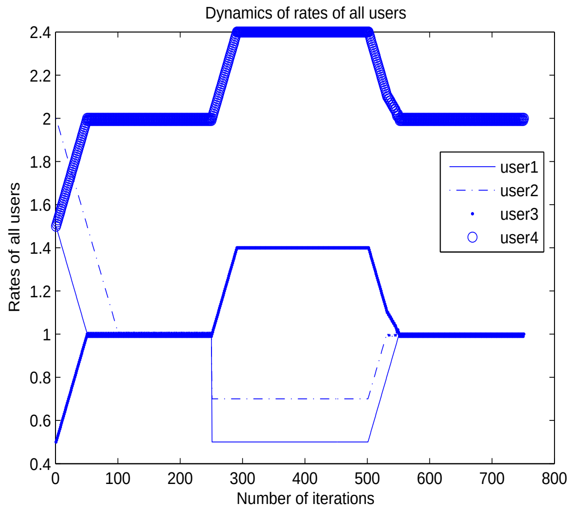


Fig. 4. Continuation of the experiment in Figures 2 and 3. At time 500, underusers 1 and 2 are again ready to send traffic at high rates. The pricing scheme ensures that the underusers reclaim their bandwidth shares while the overusers 3 and 4 back down.

This experiment illustrates that it *may* be possible to assign prices once and for all at the beginning, such that this price vector ensures fair and efficient link sharing no matter what the sets of underusers and overusers may be. Of course, we have not demonstrated that this is possible always. We are currently working on this problem.

## VI. Stability

When no user is constrained by limited amounts of available data, what is the rate vector that the collection of users converges to? When prices are set according to the interval in Equation 3 or the interval $\left(\frac{C-\gamma_i}{C}, \gamma_i\left(\frac{C\sum_{j=1, j\neq i}^{N} \lambda_j}{(\gamma_i+\sum_{j=1, j\neq i}^{N} \lambda_j)^2}+1\right)\right)$, it has been argued earlier that $(\gamma_1, \gamma_2, \ldots, \gamma_N)^t$ is the unique Nash equilibrium. Hence, when prices are set in accordance with the conditions, system stability is assured.

## VII. Conclusion

We considered sources that could occasionally be constrained by limited amounts of available data. Further, each user has a contract with the service provider, specifying the rate at which she can send traffic into the network. We were interested in a congestion-dependent as well as user-dependent pricing scheme that would ensure fair and efficient sharing of the single link shared by the sources.

We introduced the idea of disutility for underusers and noted that the disutility term encourages underusers to increase their rates whenever they have sufficient data. We presented simple necessary and sufficient conditions for setting prices such that fair and efficient sharing of the link is possible. A simple experiment in MATLAB demonstrated the utility of our approach.

We recognize the following limitations of our work.

Firstly, we considered the resource to be merely a single link; in general, of course, the resource is a full network. Secondly, we assumed a simple utility function for each user: the utility was just the carried traffic rate. We need to generalize the utility function to a concave increasing function of the carried rate. Thirdly, we have not explicitly considered the problem of revenue maximization for the service provider. While our goals of fair and efficient sharing of the link are natural and would, indirectly, lead to high revenue for the provider, we would like to consider the problem of explicit revenue maximization as well.

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## Appendix

*Proof of Lemma 3.1*

As long as user $i$ stays below her contracted rate $\gamma_i$, there is no congestion in the link and hence zero packet dropping probability. Also, the price charged by the service provider is proportional to the fraction of link capacity utilized by the user. So the net utility ($NU_i$) is,

$$NU_i = \lambda_i - e^{(\gamma_i - \lambda_i)} - \frac{\lambda_i}{\gamma_i}\pi_i$$

User $i$ has incentive to increase her rate if and only if her net utility increases with increase in $\lambda_i$. That is,

$$\begin{aligned} \frac{\mathrm{d}NU_i}{\mathrm{d}\lambda_i} &\geq 0 \\ 1 + e^{(\gamma_i - \lambda_i)} - \frac{\pi_i}{\gamma_i} &\geq 0 \end{aligned}$$

For $NU_i$ to keep increasing till user $i$ reaches $\gamma_i$, it is sufficient to have

$$\pi_i \leq 2\gamma_i$$

This condition is also necessary because if $\pi_i > 2\gamma_i$, then there exists a $\lambda_i < \gamma_i$ such that $\frac{\mathrm{d}NU_i}{\mathrm{d}\lambda_i} < 0$. This can be seen as follows. If $\pi_i > 2\gamma_i$, then $(\frac{\pi_i}{\gamma_i} - 1) > 1$. Also,

$$\frac{\mathrm{d}NU_i}{\mathrm{d}\lambda_i} = e^{(\gamma_i - \lambda_i)} - (\frac{\pi_i}{\gamma_i} - 1)$$

When $\lambda_i$ is sufficiently close to $\gamma_i$, $e^{(\gamma_i - \lambda_i)}$ drops below the fixed quantity $(\frac{\pi_i}{\gamma_i} - 1)$. At this point, $\frac{\mathrm{d}NU_i}{\mathrm{d}\lambda_i}$ becomes negative.

Since the total traffic from users $j \in \{1, 2, \ldots, N\}$, $j \neq i$, is less than their aggregate contracted rate, user $i$ should have the incentive to increase her rate further till the link is fully utilized, *i.e.*, till $\lambda_i$ is equal to $C - \sum_{j=1, j\neq i}^{N} \lambda_j$. When $\lambda_i \in [\gamma_i, C - \sum_{j=1, j\neq i}^{N} \lambda_j]$, $NU_i$ is simply $\lambda_i - \pi_i$ which is a linear increasing function of $\lambda_i$. This in turn gives user $i$ every reason to increase $\lambda_i$ in this range.

When $\lambda_i$ becomes more than $C - \sum_{j=1, j\neq i}^{N} \lambda_j$, the link gets congested. Let $\theta := \sum_{j=1, j\neq i}^{N} \lambda_j$. We would like to ensure that user $i$ has no incentive to increase her rate beyond $C - \sum_{j=1, j\neq i}^{N} \lambda_j$. Moreover, we also want to ensure that $NU_i$ decreases with $\lambda_i$ when $\lambda_i$ is greater than $(C - \theta)$. For this to happen,

$$\frac{\mathrm{d}NU_i}{\mathrm{d}\lambda_i} < 0$$

for $\lambda_i > (C - \theta)$. Let $\lambda_i = C - \theta + \delta$, for some $\delta > 0$. Then, by definition

$$NU_i = (C - \theta + \delta)(1 - \beta) - \pi_i e^{(C - \theta + \delta - \gamma_i)}$$

where

$$\begin{aligned} \beta &= \frac{(\lambda_i + \theta - C)}{(\lambda_i + \theta)} \\ &= \frac{\delta}{(C + \delta)} \end{aligned}$$

So, the requirement becomes

$$\frac{\mathrm{d}}{\mathrm{d}\delta}\left( (C - \theta + \delta)\Big(\frac{C}{C + \delta}\Big) - \pi_i e^{(C - \theta + \delta - \gamma_i)} \right) < 0$$

After some algebra, the above gives the following condition on $\pi_i$

$$\pi_i \geq \frac{C\theta}{(C + \delta)^2 e^{C - \theta - \gamma_i + \delta}}$$

Because we want the lower bound to hold for *every* $\delta > 0$, we now take the supremum of the lower bound over all $\delta > 0$. This yields

$$\pi_i \geq \frac{\theta}{C} e^{\theta - (C - \gamma_i)}$$

This proves that the lower bound on $\pi_i$ is sufficient. Necessity is shown as follows. Let $\epsilon > 0$ be a given small number. If we take $\left(\frac{\theta}{C} e^{\theta - (C - \gamma_i)} - \epsilon\right)$ as the lower bound, then simple algebra shows that there exists a $\delta > 0$ such that $\frac{\mathrm{d}NU_i}{\mathrm{d}\lambda_i} > 0$. This contradicts our requirement and concludes the proof.

□